Kapasitansi dan Dielektrik
*(Capacitance and Dielectrics)*

Halliday-Resnick-Walker, Fundamentals of Physics
The **capacitance** $C$ of a capacitor is the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them:

$$C \equiv \frac{Q}{\Delta V}$$  \hspace{1cm} (26.1)

Satuan : $1 \text{F} = 1 \frac{C}{V}$
Kapasitansi Keping Sejajar:

$$\Delta V = \int \mathbf{E} \cdot d\mathbf{r} = \int_0^d E_x \, dx = E_x d$$

Medan listrik diantara kedua keeping konduktor:

$$E_x = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{E_x d}$$

$$C = \frac{\varepsilon_0 A}{d}$$
Parallel-Plate Capacitors

\[ E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \]

\[ \Delta V = Ed = \frac{Qd}{\varepsilon_0 A} \]

\[ C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\varepsilon_0 A} \]

\[ C = \frac{\varepsilon_0 A}{d} \]
The Cylindrical Capacitor

\[ \Delta V = \int_{a}^{b} E(r) \, dr \quad E(2\pi rL) = \frac{Q}{\varepsilon_0} \quad E = \frac{Q}{2\pi \varepsilon_0 L r} \]

\[ \Delta V = \int_{a}^{b} \frac{Q}{2\pi \varepsilon_0 L r} \, dr = \frac{Q}{2\pi \varepsilon_0 L} \ln(b/a) \]

Kapasitansinya adalah:

\[ C = \frac{Q}{\Delta V} = \frac{Q}{\frac{2\pi \varepsilon_0 L}{\ln(b/a)}} \quad \text{dan} \quad C = \frac{2\pi \varepsilon_0 L}{\ln(a/b)} \]
Kapasitansi Dua Konduktor Lingkaran Konsentrik

\[ E_r = k \frac{Q}{r^2} \quad a < r < b \]

\[ \Delta V = V_a - V_b = \int_a^b E_r \, dr \]

\[ \Delta V = kQ \int_a^b \frac{dr}{r^2} = kQ \left( \frac{1}{a} - \frac{1}{b} \right) \]

\[ \Delta V = \frac{kQ(b - a)}{ab} \]

\[ C = \frac{Q}{\Delta V} = \frac{ab}{k(b - a)} = \frac{4\pi \varepsilon_0 ab}{(b - a)} \]
The Spherical Capacitor

\[ V_b - V_a = - \int_a^b E_r \, dr = -k_e Q \int_a^b \frac{dr}{r^2} = k_e Q \left[ \frac{1}{r} \right]_a^b \]

\[ = k_e Q \left( \frac{1}{b} - \frac{1}{a} \right) \]

\[ \Delta V = |V_b - V_a| = k_e Q \frac{(b - a)}{ab} \]

\[ C = \frac{Q}{\Delta V} = \frac{ab}{k_e(b - a)} \]
COMBINATIONS OF CAPACITORS

Parallel Combination

\[ Q = Q_1 + Q_2 \]

\[ C_{eq} \Delta V = C_1 \Delta V + C_2 \Delta V \]

\[ C_{eq} = C_1 + C_2 \] (parallel combination)
COMBINATIONS OF CAPACITORS

Series Combination

\[ \Delta V = \Delta V_1 + \Delta V_2 \]

\[ \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} \]

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \]

(series combination)
ENERGY STORED IN A CHARGED CAPACITOR

This bank of capacitors stores electrical energy for use in the particle accelerator at FermiLab, located outside Chicago.

\[ dW = \Delta V \, dq = \frac{q}{C} \, dq \]

\[ W = \int_0^Q \frac{q}{C} \, dq = \frac{1}{C} \int_0^Q q \, dq = \frac{Q^2}{2C} \]

\[ U = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 \]
CAPACITORS WITH DIELECTRICS

\[ C = \kappa C_0 \]

\[ C = \kappa \frac{\varepsilon_0 A}{d} \]
<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric Constant $\kappa$</th>
<th>Dielectric Strength(^a) (V/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air (dry)</td>
<td>1.000 59</td>
<td>$3 \times 10^6$</td>
</tr>
<tr>
<td>Bakelite</td>
<td>4.9</td>
<td>$24 \times 10^6$</td>
</tr>
<tr>
<td>Fused quartz</td>
<td>3.78</td>
<td>$8 \times 10^6$</td>
</tr>
<tr>
<td>Neoprene rubber</td>
<td>6.7</td>
<td>$12 \times 10^6$</td>
</tr>
<tr>
<td>Nylon</td>
<td>3.4</td>
<td>$14 \times 10^6$</td>
</tr>
<tr>
<td>Paper</td>
<td>3.7</td>
<td>$16 \times 10^6$</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.56</td>
<td>$24 \times 10^6$</td>
</tr>
<tr>
<td>Polyvinyl chloride</td>
<td>3.4</td>
<td>$40 \times 10^6$</td>
</tr>
<tr>
<td>Porcelain</td>
<td>6</td>
<td>$12 \times 10^6$</td>
</tr>
<tr>
<td>Pyrex glass</td>
<td>5.6</td>
<td>$14 \times 10^6$</td>
</tr>
<tr>
<td>Silicone oil</td>
<td>2.5</td>
<td>$15 \times 10^6$</td>
</tr>
<tr>
<td>Strontium titanate</td>
<td>233</td>
<td>$8 \times 10^6$</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
<td>$60 \times 10^6$</td>
</tr>
<tr>
<td>Vacuum</td>
<td>1.000 00</td>
<td>—</td>
</tr>
<tr>
<td>Water</td>
<td>80</td>
<td>—</td>
</tr>
</tbody>
</table>

\(^a\) The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.
Types of Capacitors

(a) Metal foil
   Paper

(b) Plates
    Oil

(c) Case
   Metallic foil + oxide layer
   Electrolyte
   Contacts
AN ATOMIC DESCRIPTION OF DIELECTRICS

(a) Polar molecules are randomly oriented in the absence of an external electric field. (b) When an external field is applied, the molecules partially align with the field.
AN ATOMIC DESCRIPTION OF DIELECTRICS

\[ E = \frac{E_0}{\kappa} \]

\[ E = E_0 - E_{\text{ind}} \]

\[ \frac{\sigma}{\kappa\varepsilon_0} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_{\text{ind}}}{\varepsilon_0} \]

\[ \sigma_{\text{ind}} = \left( \frac{\kappa - 1}{\kappa} \right) \sigma \]
Effect of a Metallic Slab

A parallel-plate capacitor has a plate separation $d$ and plate area $A$. An uncharged metallic slab of thickness $a$ is inserted midway between the plates. Find the capacitance of the device.

\[
\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\epsilon_0 A \left(\frac{d-a}{2}\right)} + \frac{1}{\epsilon_0 A \left(\frac{d-a}{2}\right)}
\]

\[
C = \frac{\epsilon_0 A}{d-a}
\]

\[
C = \lim_{a \to 0} \frac{\epsilon_0 A}{d-a} = \frac{\epsilon_0 A}{d}
\]
Direct Current Circuits

Electromotive Force

\[ \Delta V = \mathcal{E} - Ir \]
\[ \mathcal{E} = IR + Ir \]

\[ I = \frac{\mathcal{E}}{R + r} \]
RESISTORS IN SERIES AND IN PARALLEL

\[ R_{eq} = R_1 + R_2 + R_3 + \cdots \]
RESISTORS IN SERIES AND IN PARALLEL

(a) Two resistors $R_1$ and $R_2$ in parallel.
(b) A series connection of resistors $R_1$ and $R_2$.
(c) Equivalent resistance $R_{eq}$.

\[
\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots
\]
Charging a Capacitor

(a) 

(b) $t < 0$

(c) $t > 0$
\[ V - \frac{Q}{C} - iR = 0 \]

dengan menggunakan:
\[ i = \frac{dq}{dt} \]

\[ -\frac{1}{C} i - R \frac{di}{dt} = 0 \quad \quad \frac{di}{i} = -\frac{1}{RC} dt \]

\[ i = \frac{V}{R} e^{-r/RC} = i_0 e^{-t/RC} \]

dari persamaan *)

\[ Q(t) = CV (1 - e^{-t/RC}) \]
dari persamaan *)

\[ Q(t) = CV \left( 1 - e^{-t/RC} \right) \]
\[ e = 2.71828182845904523536028747135266249775724709369995... \]
Discharging a Capacitor

\[
\frac{Q}{C} - iR = 0 \quad \frac{1}{C} \frac{dQ}{dt} - R \frac{di}{dt} = 0
\]

- arus dalam rangkaian sama dengan laju penu-runan muatan \( Q \) di \( C \), sehingga:

\[
i = -\frac{dQ}{dt}, \quad \frac{di}{dt} = -\frac{i}{RC}
\]

\[
\frac{di}{i} = -\frac{1}{RC} \, dt \quad \int_{i=\Delta V_0/R}^{i=i_f} \frac{di}{i} = -\frac{1}{RC} \int_{t=0}^{t=t_f} dt
\]

\[
i = \frac{\Delta V_0}{R} e^{-r/RC} = i_0 e^{-t/RC}
\]
Discharging a Capacitor

\[ i_0 = \frac{\Delta V_0}{R} \]

\[ 0.37 \, i_0 \]

\[ \tau = RC \]

\[ i = \frac{\Delta V_0}{R} e^{-r/RC} = i_0 e^{-t/RC} \]