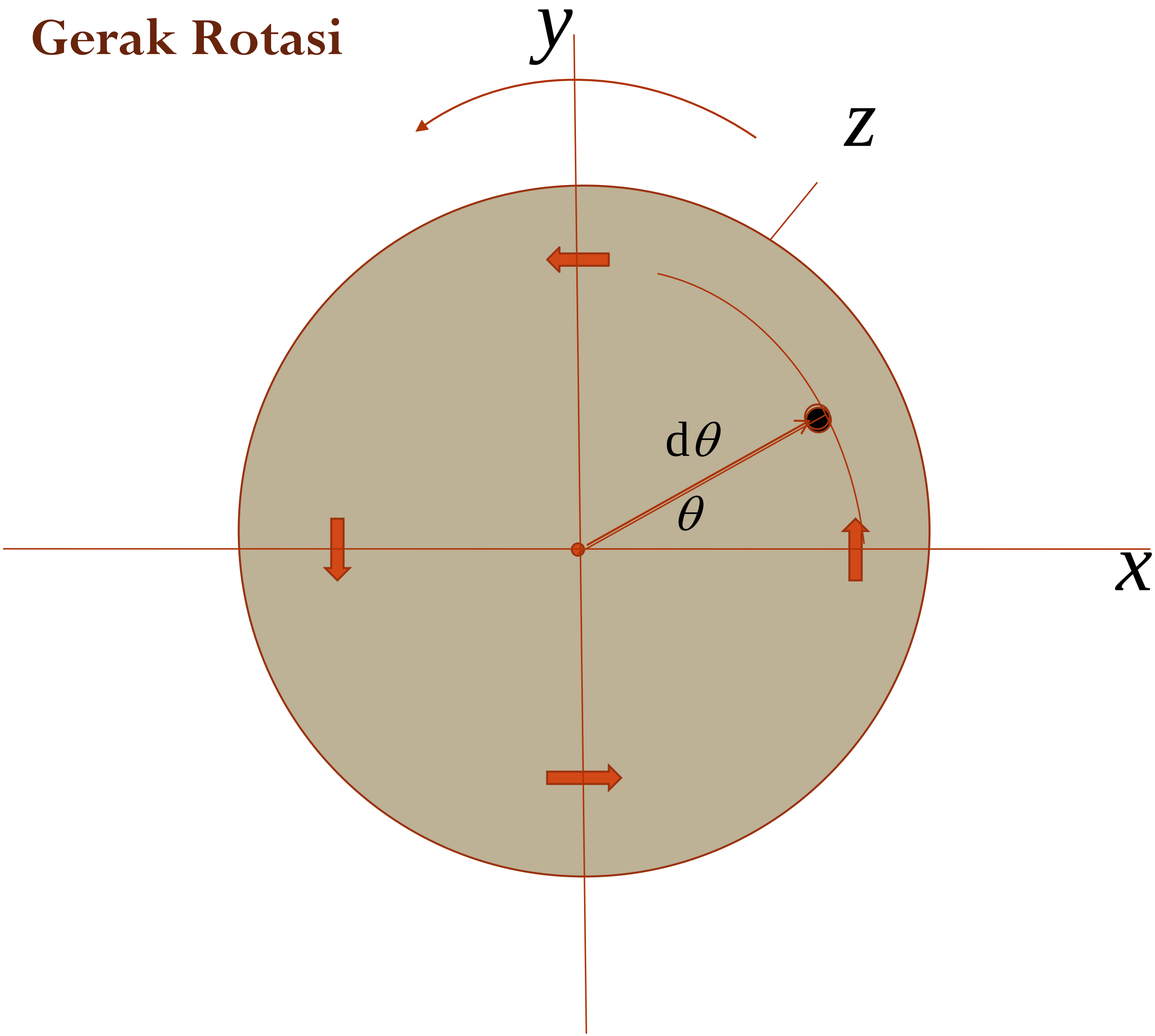
Gerak Rotasi
(*Rotational Motion*)

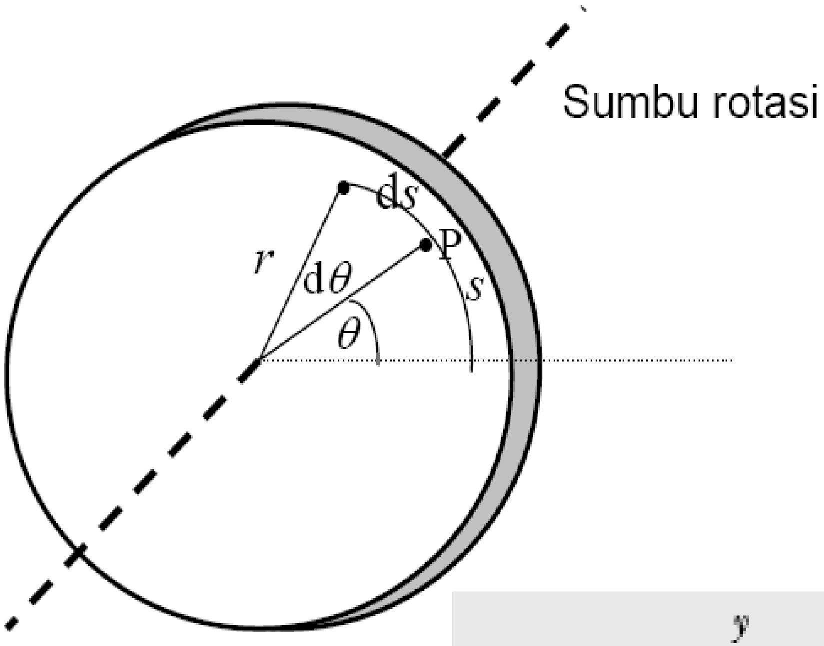
Analisis Gerak Rotasi

- Kinematika rotasi (*rotational kinematics)*
- Energitika rotasi (*rotational energetics)*
- Dinamika Rotasi (*rotational dynamics)*

Gerak Rotasi



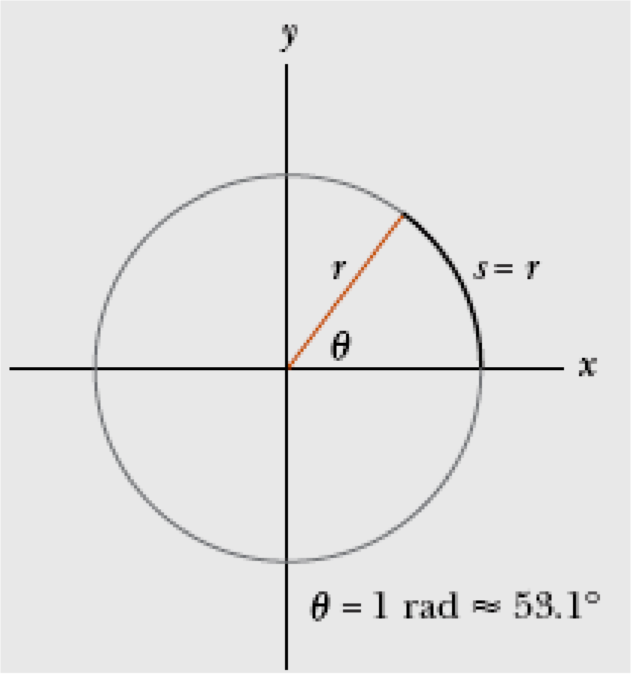
Kinematika Rotasi



$$v_t = \frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$\omega = \frac{d\theta}{dt}$$

$$s = r\theta$$



$$v_t = r\omega$$

Percepatan Tangensial dan Percepatan Sudut

$$\frac{\mathrm{d}v_t}{\mathrm{d}t} = r\frac{\mathrm{d}\omega}{\mathrm{d}t} \qquad a_t = \frac{\mathrm{d}v_t}{\mathrm{d}t}$$

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} \qquad a_t = r\alpha$$

- Persamaan Kinematika Rotasi untuk Percepatan Sudut Konstan adalah:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \qquad \omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$$

Contoh Soal 1:

Pengendara sepeda memperlambat gerak sepedanya secara beraturan dari v_0=8,40 m/s sampai berhenti dalam jarak 115 m. Roda dan ban sepeda memiliki diameter keseluruhan 68,0 cm. Tentukan (a) kecepatan sudut roda pada saat awal, (b) jumlah total putaran setiap roda sampai berhenti, (c) percepatan sudut roda, dan (d) waktu yang diperlukan sampai berhenti.

Jawaban:

(a) $$\omega_0 = \frac{v_0}{r} = \frac{8{,}40 \text{ m/s}}{0{,}340 \text{ m}} = 24{,}7 \text{ rad/s}$$

(b) Setiap putaran berhubungan dengan jarak $2\pi r$, sehingga :

$$\frac{115 \text{ m}}{2\pi r} = \frac{115 \text{ m}}{(2\pi)(0{,}340 \text{ m})} = 53{,}8 \text{ putaran}$$

(c) $$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta} = \frac{0 - (24{,}7 \text{ rad/s})^2}{2(2\pi)(53{,}8 \text{ put})} = -0{,}902 \text{ rad/s}^2$$

(d) $$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 24{,}7 \text{ rad/s}}{-0{,}902 \text{ rad/s}^2} = 27{,}4 \text{ s}$$

(Giancoli, 1999)

Contoh Soal 2:

- Radius rata-rata bumi 6,38 x 10^3 km. Hitung (a) kecepatan sudut rotasi bumi, (b) kecepatan linear orang yang betempat tinggal di garis katulistiwa, (c) kecepatan linear orang yang tinggal pada garis lintang 23^0.

Jawaban:

$$\omega_{\text{bumi}} = \frac{\theta}{t} = \frac{2\pi}{24\ \text{jam}}$$

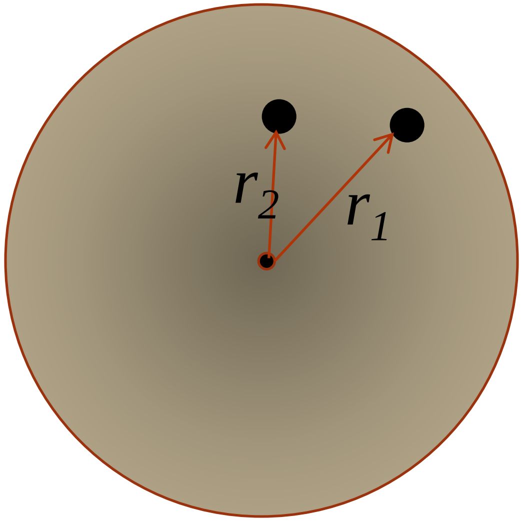
$$v_{\text{objek di katulistiwa}} = R\omega = 6,38 \times 10^3\,\text{km} \times \frac{2\pi}{24\ \text{jam}}$$

$$v_{\text{objek di katulistiwa}} = 1,6485 \times 10^3\ \text{km/jam} = 1.648,5\ \text{km/jam}$$

Energi Rotasi dan Momen Inersia
(*Rotational Energy and Moment of Inertia*)

- Energi kinetik dari masing-masing partikel penyusun benda tegar adalah:

$$E_k = \frac{1}{2} m v_t^{\,2} = \frac{1}{2} m r^2 \omega^2$$



Besaran mr^2 didefinisikan sebagai **momen inersia** partikel terhadap sumbu rotasi, atau:

$$I = mr^2 \qquad E_k = \frac{1}{2} I \omega^2$$

Untuk benda tegar yang tersusun oleh beberapa partikel individual bermassa m_1, m_2 , …...pada jarak tetap r_1, r_2, .. dari sumbu rotasi tetap, maka energi kinetik benda tegar adalah:

$$E_k = \frac{1}{2}\left(m_1 r_1^{\,2}\omega^2 + m_2 r_2^{\,2}\omega^2 + \cdots\right) = \frac{1}{2}\left(\sum_i m_i r_i^{\,2}\right)\omega^2$$

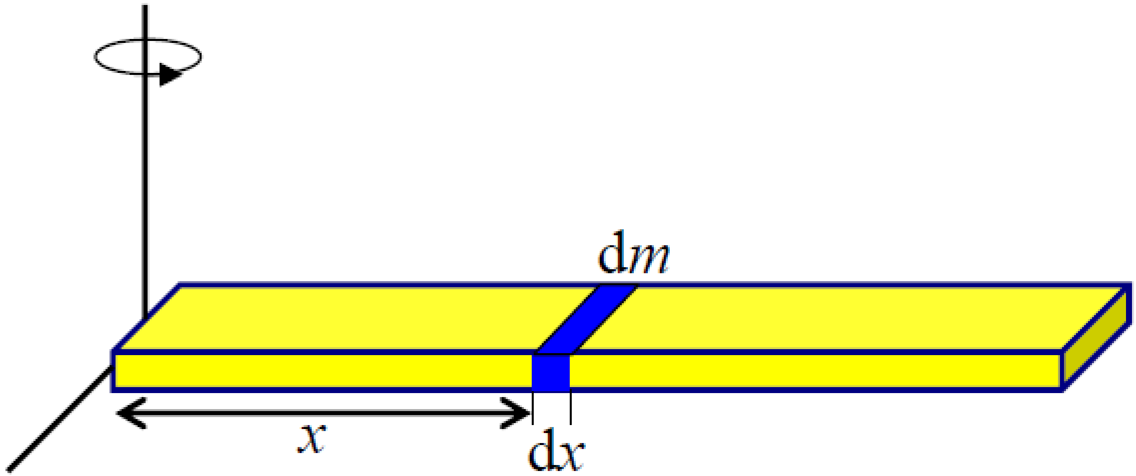
- Momen inersia untuk benda tegar adalah:

$$I = \sum_i m_i r_i^2$$

Untuk benda tegar yang tersusun dari massa berdistribusi kontinyu, momen inersia terhadap sumbu yang dipilih adalah:

$$I = \int_{benda} r^2 \mathrm{d}m$$

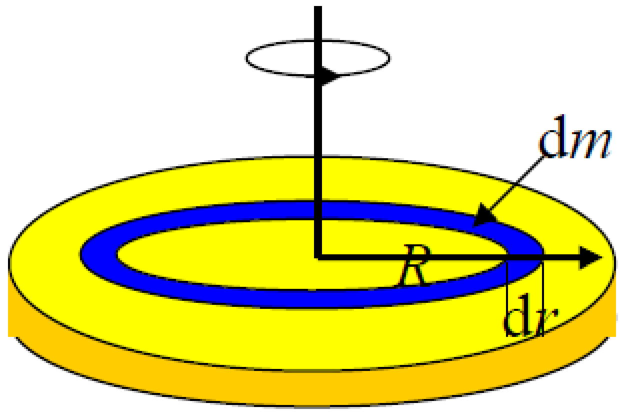
- Momen Inersia Batang Uniform yang diputar pada sumbu tegak di salah satu ujung batang



$$I_y = \int_0^L x^2 dm = \int_0^L x^2 \frac{M}{L} dx$$

$$I_y = \frac{M}{L}\frac{1}{3}x^3 = \frac{ML^3}{3L} = \frac{1}{3}ML^2$$

- Momen Inersia Cakram Uniform yang diputar terhadap sumbu tegak lurus bidang cakram di pusat cakram



$$dm = \frac{M}{A} dA = \frac{M}{A} 2\pi r dr$$

$$I = \int r^2 dm = \int_0^R r^2 \frac{M}{A} 2\pi r dr$$

$$I = \frac{2\pi M}{\pi R^2} \int_0^R r^3 dr = \frac{1}{2} MR^2$$

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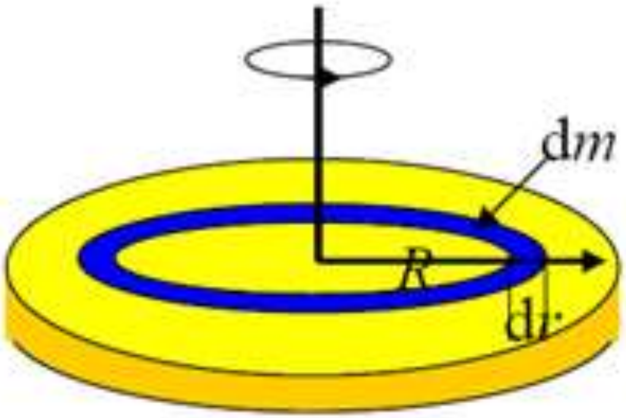

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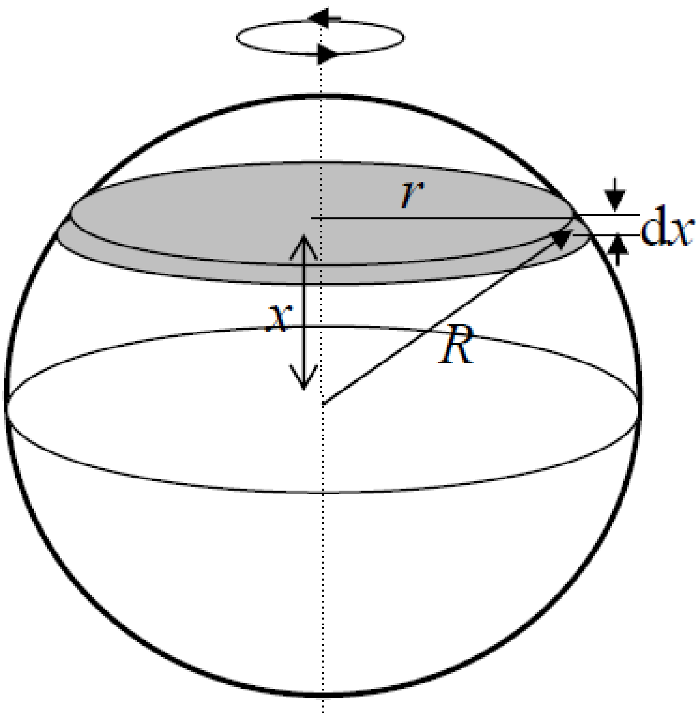
Momen Inersia Bola Uniform yang berputar pada diameternya



$$r = \sqrt{R^2 - x^2}$$

$$dm = \frac{M}{V} dV = \frac{M}{V} \pi r^2 dx = \frac{M}{V} \pi (R^2 - x^2) dx$$

$$dI = \frac{1}{2} r^2 dm = \frac{1}{2} \frac{M}{V} \pi (R^2 - x^2)^2 dx$$

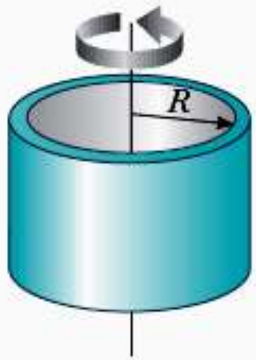
$$I = 2\int_0^R \frac{1}{2} \frac{M}{V} \pi (R^2 - x^2)^2 dx$$

$$I = \frac{\pi M}{V} \frac{8R^5}{15} = \frac{2}{5} MR^2$$

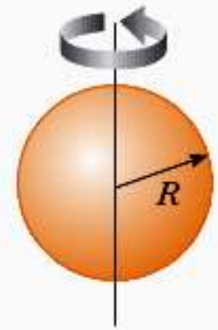
TABLE 8.1

Moments of Inertia for Various Rigid Objects of Uniform Composition

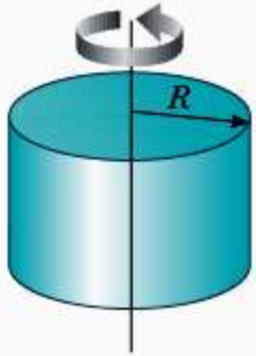
Hoop or thin cylindrical shell
$I = MR^2$



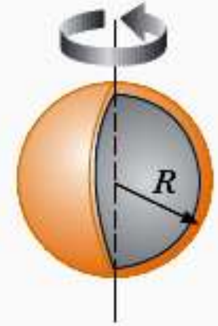
Solid sphere
$I = \frac{2}{5} MR^2$



Solid cylinder or disk
$I = \frac{1}{2} MR^2$

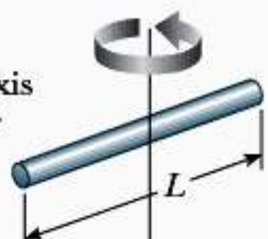


Thin spherical shell
$I = \frac{2}{3} MR^2$



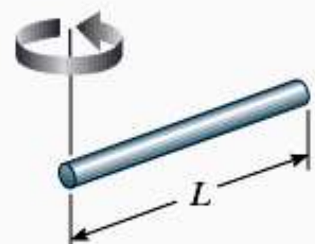
Long thin rod with rotation axis through center

$I = \frac{1}{12} ML^2$



Long thin rod with rotation axis through end

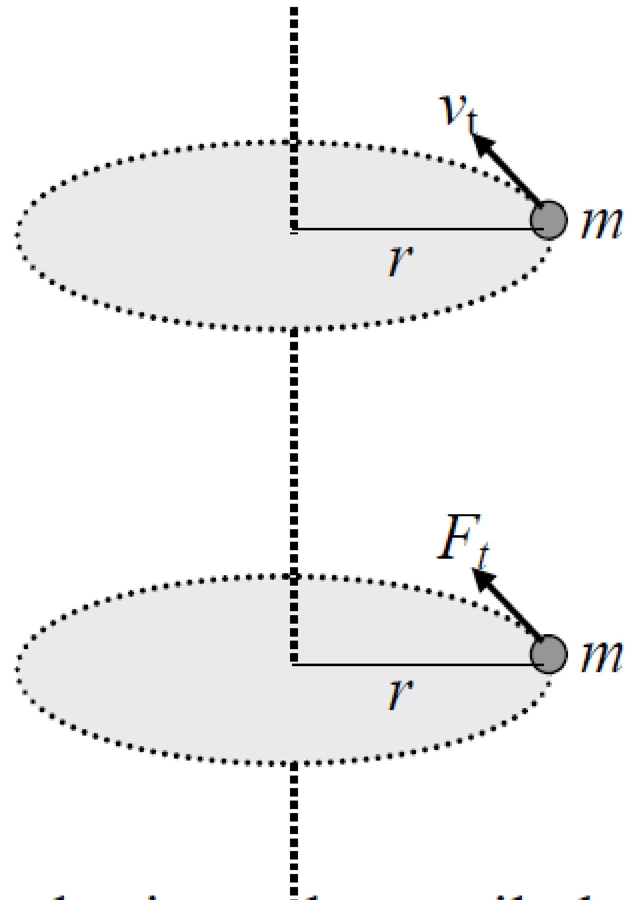
$I = \frac{1}{3} ML^2$



Momentum Sudut (*Angular Momentum*)

$$\mathbf{L} = \mathbf{r} \times m\mathbf{v} = \mathbf{r} \times \mathbf{p}$$

$$L = rmv_t = rp_t$$



Torsi (*Torque*)

$$\tau = \mathbf{r} \times \mathbf{F}$$

$$\tau = F_t r$$

Jika ada beberapa gaya yang berkerja pada partikel maka torsi total yang bekerja pada partikel itu adalah:

Jika ada beberapa gaya yang berkerja pada partikel maka torsi total yang bekerja pada partikel itu adalah:

$$\sum \tau = F_{t1} r + F_{t2} r + \cdots = r \sum F_t$$

$$\sum \tau = r \frac{\mathrm{d} p_t}{\mathrm{d} t} \quad \sum \tau = \frac{\mathrm{d} L}{\mathrm{d} t}$$

$$L = r p_t = r m v_t = m r^2 \omega$$

$$L = I \omega$$

Dinamika Rotasi (*Rotational Dynamics*)

$$\sum \tau = \frac{\mathrm{d}L}{\mathrm{d}t} = \frac{\mathrm{d}(I\omega)}{\mathrm{d}t} = I\,\frac{\mathrm{d}\omega}{\mathrm{d}t}$$

$$\sum \tau = I\alpha$$

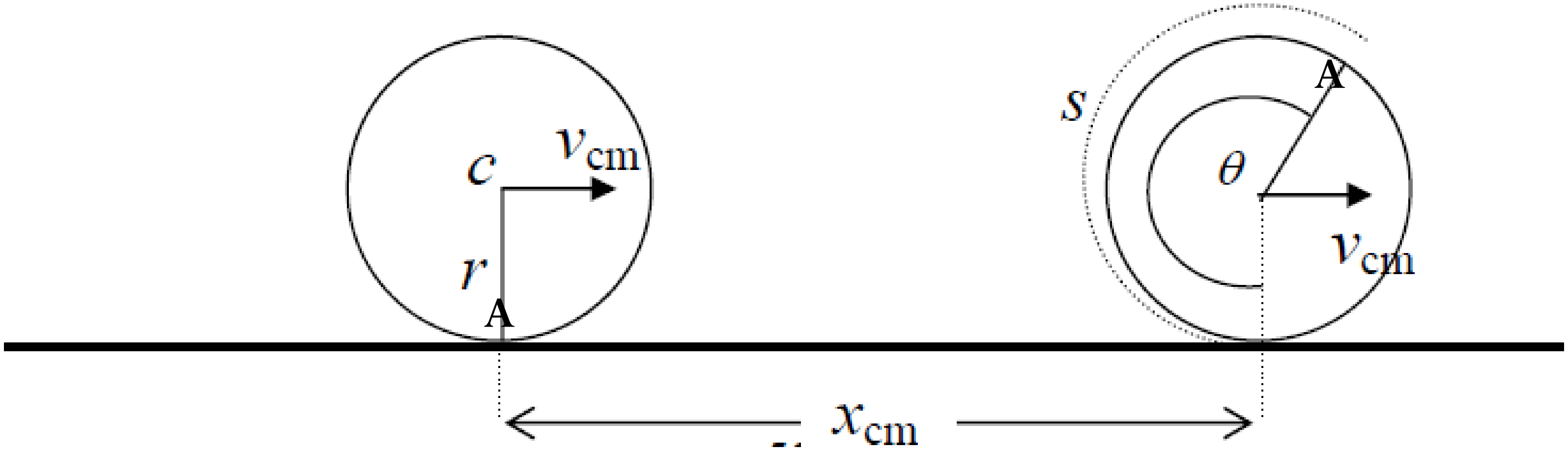
Kekekalan Momentum Sudut

- Jika torsi total pada system partikel sama dengan nol, maka:

$$\sum \tau = \frac{\mathrm{d}L}{\mathrm{d}t} = 0 \qquad L_i = L_f$$

$$I_i\omega_i = I_f\omega_f$$

Gerak Simultan Rotasi dan Translasi

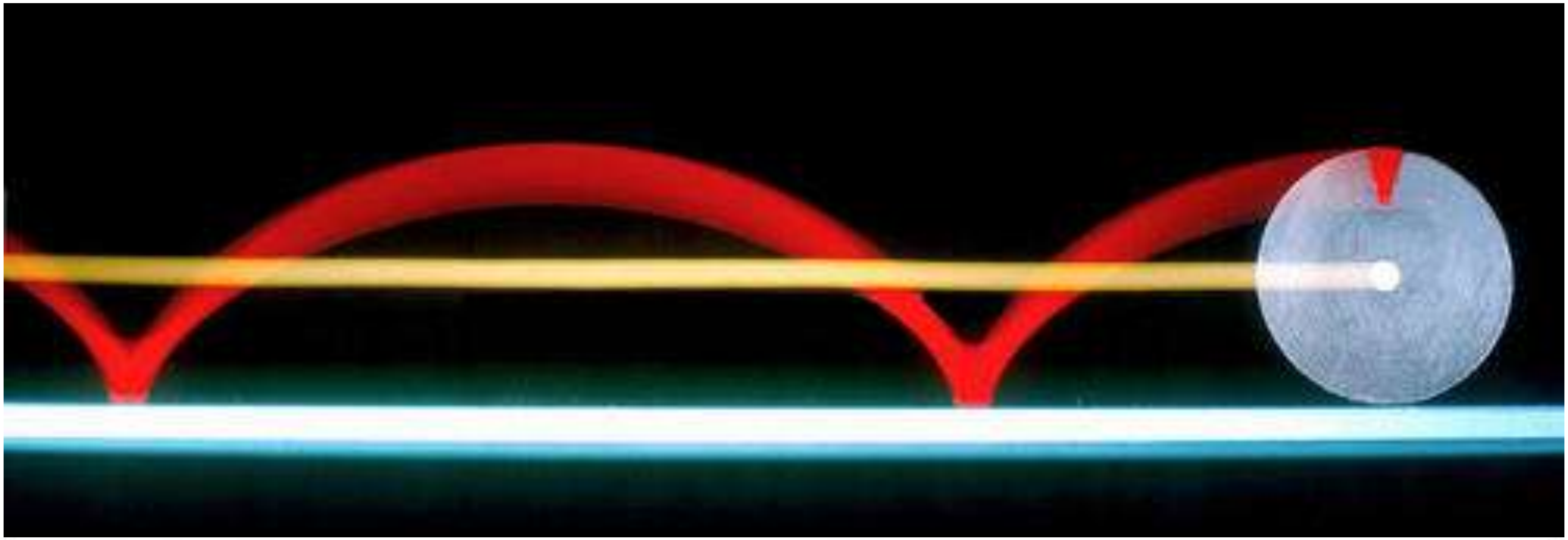


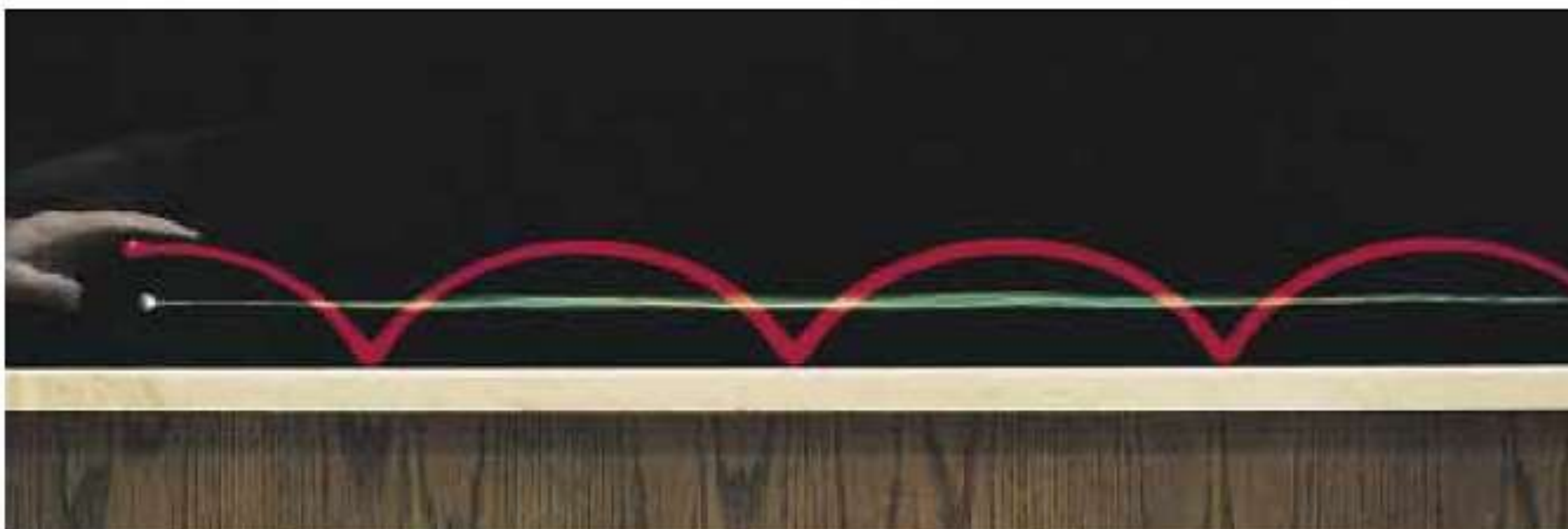
Posisi pusat massa: : $x_{cm} = s = r\theta$

Kecepatan pusat massa : $v_{cm} = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$

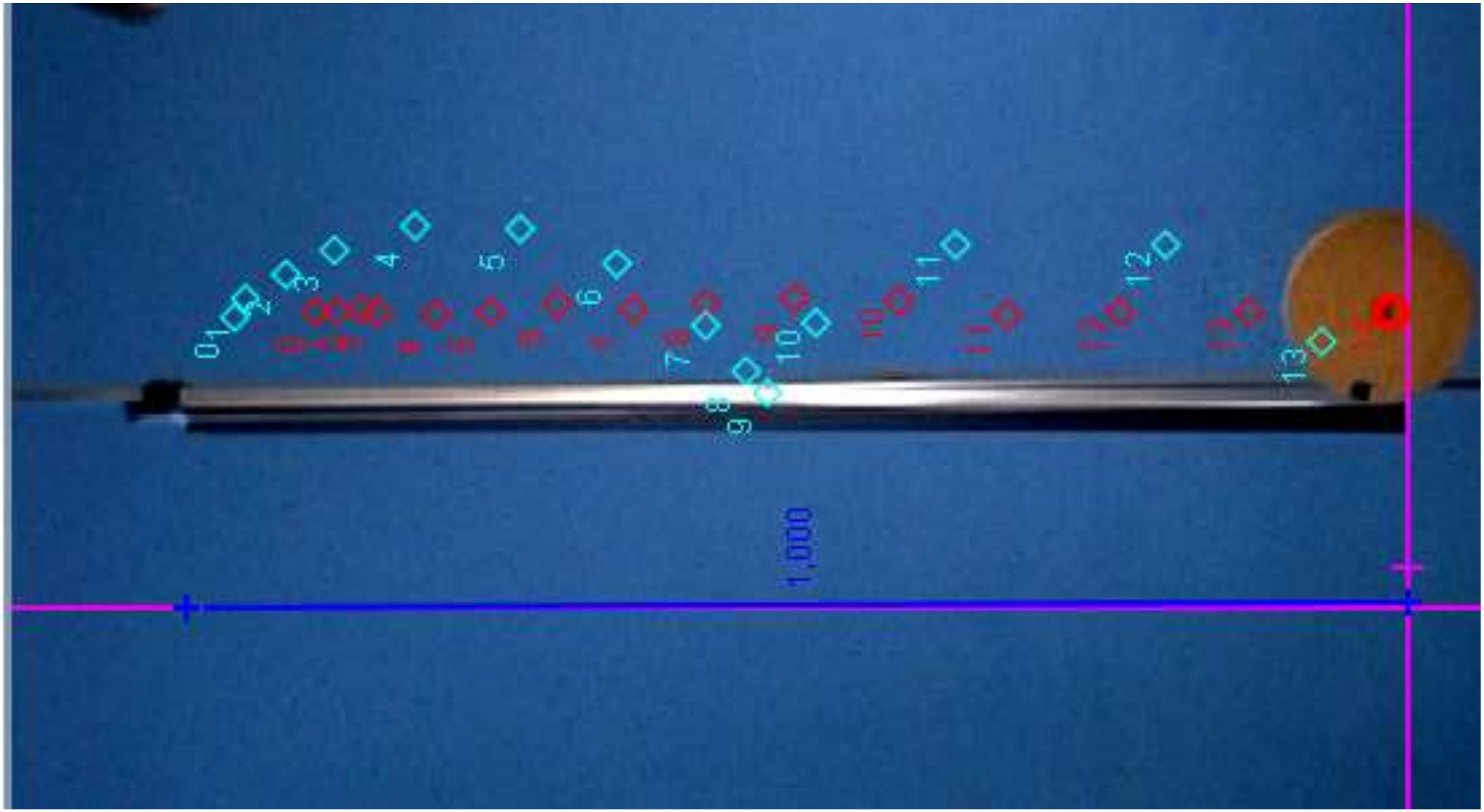
Percepatan pusat massa : $a_{cm} = \frac{dv_{cm}}{dt} = r\frac{d\omega}{dt} = r\alpha$

Gerak Simultan Rotasi dan Translasi

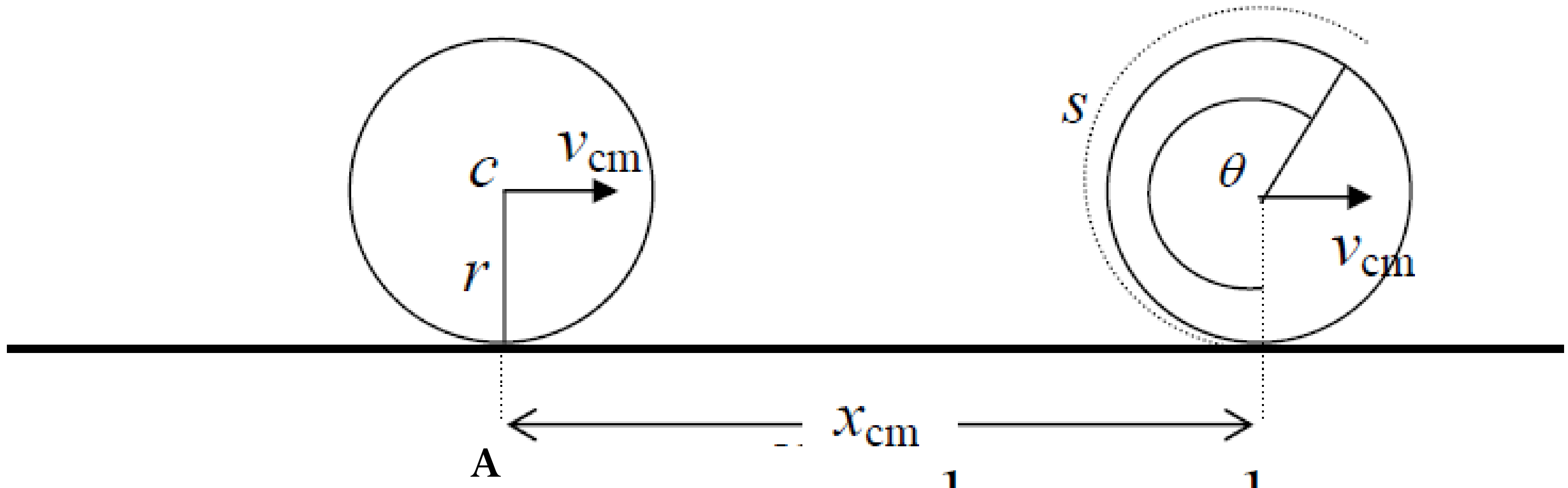




Gerak Simultan Rotasi dan Translasi



Gerak Simultan Rotasi dan Translasi



$$K = \frac{1}{2} I_A \omega^2$$

$$I_A = I_{cm} + MR^2$$

$$K = \frac{1}{2}(I_{cm} + MR^2)\omega^2$$

$$K = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} MR^2\omega^2$$

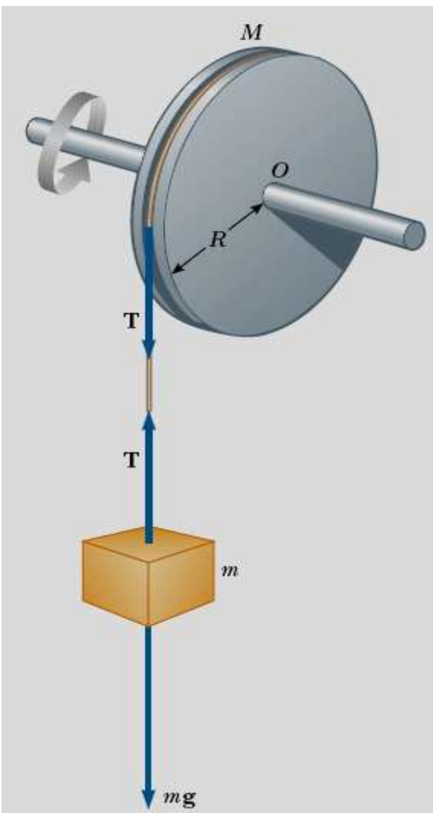
$$K = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} Mv_{cm}^2$$

Energi Kinetik Rotasi

Energi Kinetik Translasi

Contoh Soal 4:

- **Angular Acceleration of a Wheel.** A wheel of radius *R*, mass *M*, and moment of inertia *I* is mounted on a frictionless, horizontal axle, as shown in Figure. A light cord wrapped around the wheel supports an object of mass *m*. Calculate the angular acceleration of the wheel, the linear acceleration of the object, and the tension in the cord.



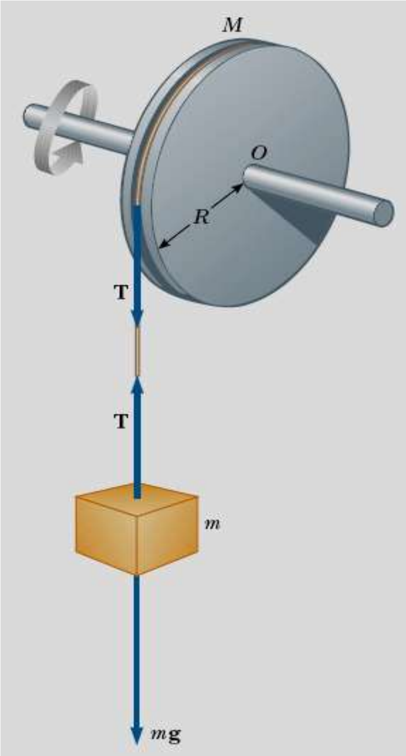
$$\sum \tau = I\alpha = TR \qquad \alpha = \frac{TR}{I}$$

$$\sum F_y = mg - T = ma \qquad a = \frac{mg - T}{m}$$

$$a = R\alpha = \frac{TR^2}{I} = \frac{mg - T}{m} \qquad T = \frac{mg}{1 + \frac{mR^2}{I}}$$

$$a = \frac{g}{1 + I/mR^2} \qquad \alpha = \frac{a}{R} = \frac{g}{R + I/mR}$$

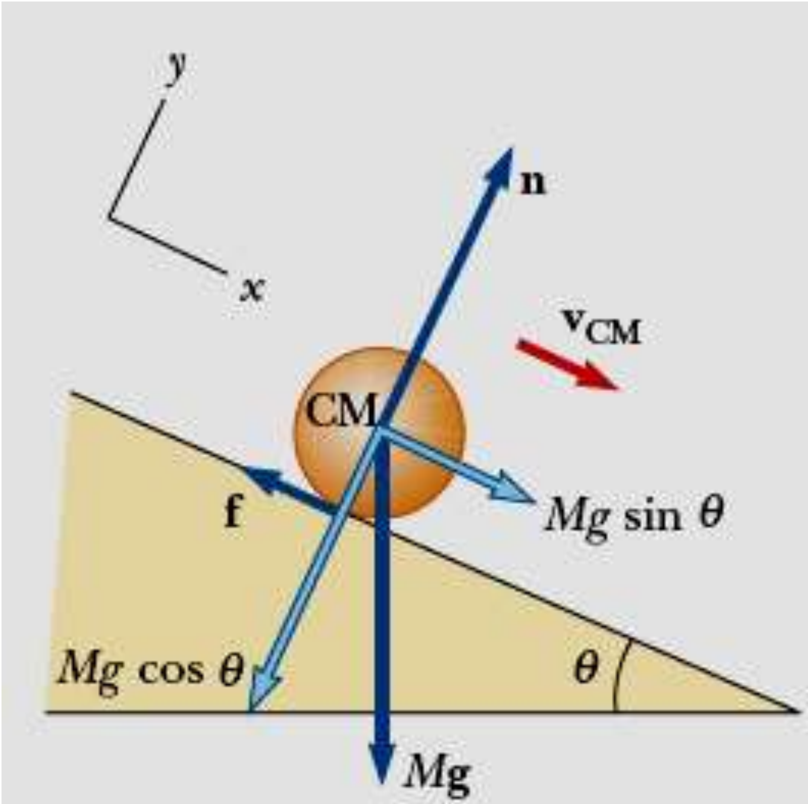
- The wheel in Figure is a solid disk of M =2.00 kg, R =30.0 cm, and I= 0.090 kg.m^2. The suspended object has a mass of m=0.500 kg. Find the tension in the cord and the angular acceleration of the wheel.



$$T = \frac{mg}{1 + \frac{mR^2}{I}} \qquad \alpha = \frac{a}{R} = \frac{g}{R + I/mR}$$

Answer : 3.27 N; 10.9 rad/s^2.

Another Look at the Rolling Sphere



Newton's second law applied to the center of mass gives

$$\sum F_x = Mg\sin\theta - f = Ma_{CM}$$

$$\sum F_y = n - Mg\cos\theta = 0$$

$$\sum \tau_{CM} = I_{CM}\alpha = fR$$

$$I_{CM} = \frac{2}{5}MR^2 \qquad \alpha = \frac{a_{CM}}{R}$$

$$f = \frac{I_{CM}\alpha}{R} = \left(\frac{\frac{2}{5}MR^2}{R}\right)\frac{a_{CM}}{R} = \frac{2}{5}Ma_{CM} \qquad a_{CM} = \frac{5}{7}g\sin\theta$$

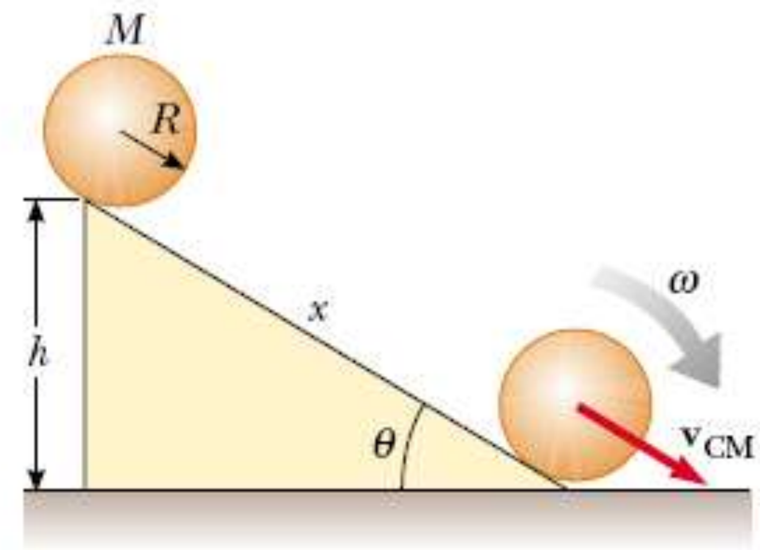
Total kinetic energy of a rolling body

$$K = \tfrac{1}{2} I_{CM} \omega^2 + \tfrac{1}{2} M v_{CM}{}^2$$

$$v_{CM} = R\omega$$

$$K = \tfrac{1}{2} I_{CM} \left(\frac{v_{CM}}{R} \right)^2 + \tfrac{1}{2} M v_{CM}{}^2$$

$$K = \tfrac{1}{2} \left(\frac{I_{CM}}{R^2} + M \right) v_{CM}{}^2$$

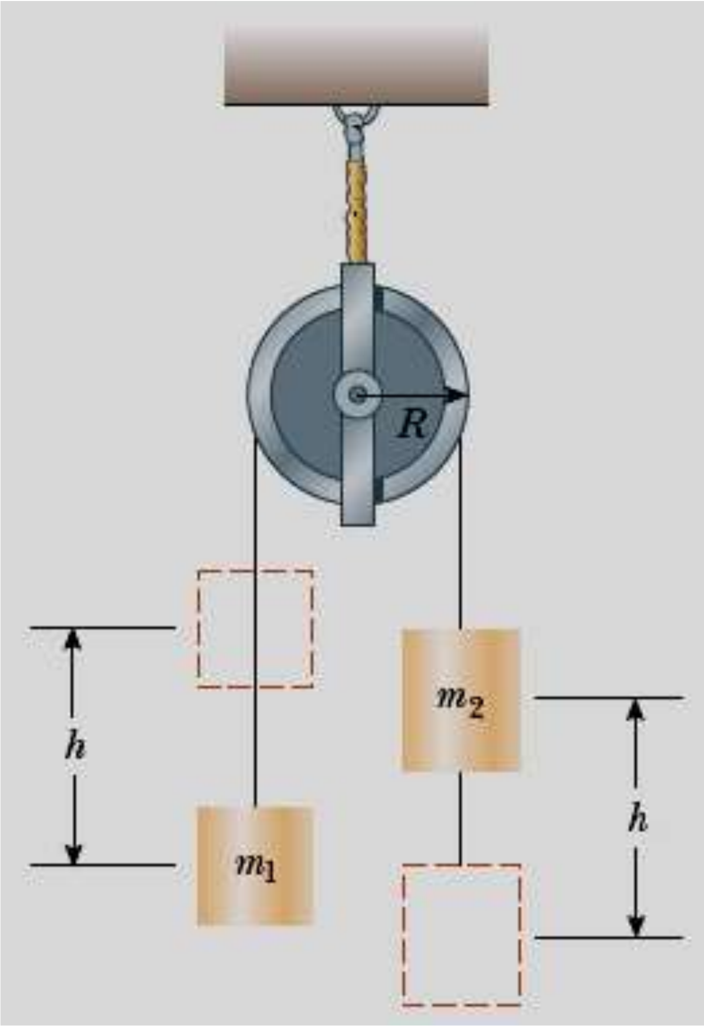


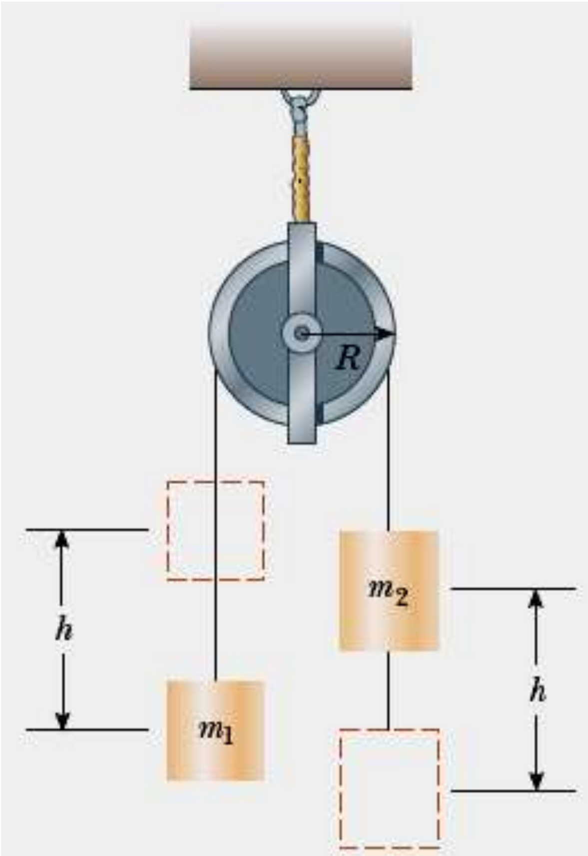
$$\tfrac{1}{2} \left(\frac{I_{CM}}{R^2} + M \right) v_{CM}{}^2 = Mgh$$

$$v_{CM} = \left(\frac{2gh}{1 + I_{CM}/MR^2} \right)^{1/2}$$

Connected Cylinders

- Consider two cylinders having masses m_1 *and* m_2*, where* $m_1 \neq m_2$*, connected by a string passing over a pulley, as shown in* Figure . The pulley has a radius *R and moment of* inertia *I about its axis of rotation. The string does not slip* on the pulley, and the system is released from rest. Find the linear speeds of the cylinders after cylinder 2 descends through a distance *h, and the angular speed of the pulley at* this time.





Because $K_i = 0$ (the system is initially at rest), we have:

$$\Delta K = K_f - K_i = (\frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 + \frac{1}{2} I\omega_f^2) - 0$$

where v_f is the same for both blocks. Because $v_f = R\omega_f$, this expression becomes:

$$\Delta K = \frac{1}{2}(m_1 + m_2 + \frac{I}{R^2})v_f^2$$

we see that the system loses potential energy as cylinder 2 descends and gains potential energy as cylinder 1 rises

$$\Delta U_2 = -m_2 gh \qquad \Delta U_1 = m_1 gh$$

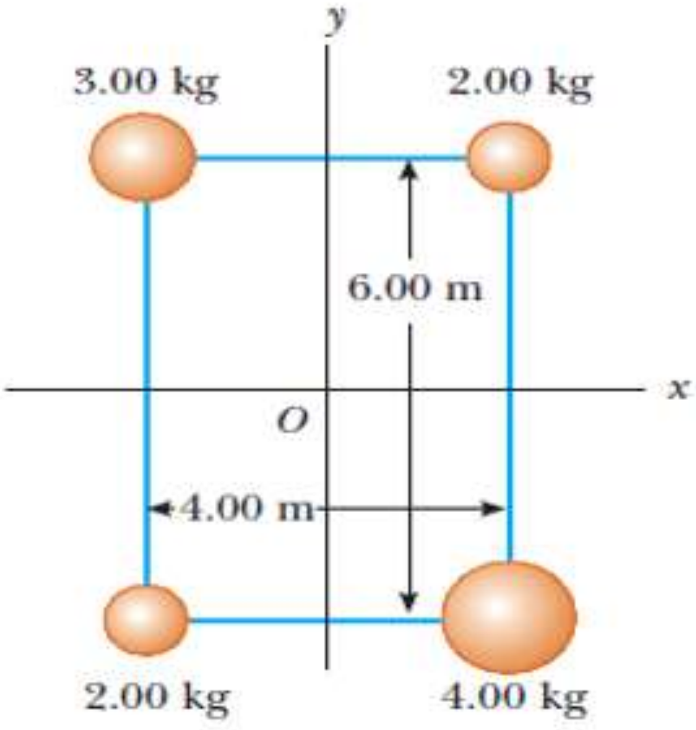
Applying the principle of conservation of energy in the form

$$\Delta K + \Delta U_1 + \Delta U_2 = 0 \qquad \frac{1}{2}(m_1 + m_2 + \frac{I}{R^2})v_f^2 + m_1 gh - m_2 gh = 0$$

$$v_f = \left[\frac{2(m_2 - m_1)gh}{\left(m_1 + m_2 + \frac{I}{R^2} \right)} \right]^{\frac{1}{2}} \qquad \omega_f = \frac{v_f}{R} = \frac{1}{R}\left[\frac{2(m_2 - m_1)gh}{\left(m_1 + m_2 + \frac{I}{R^2} \right)} \right]^{\frac{1}{2}}$$

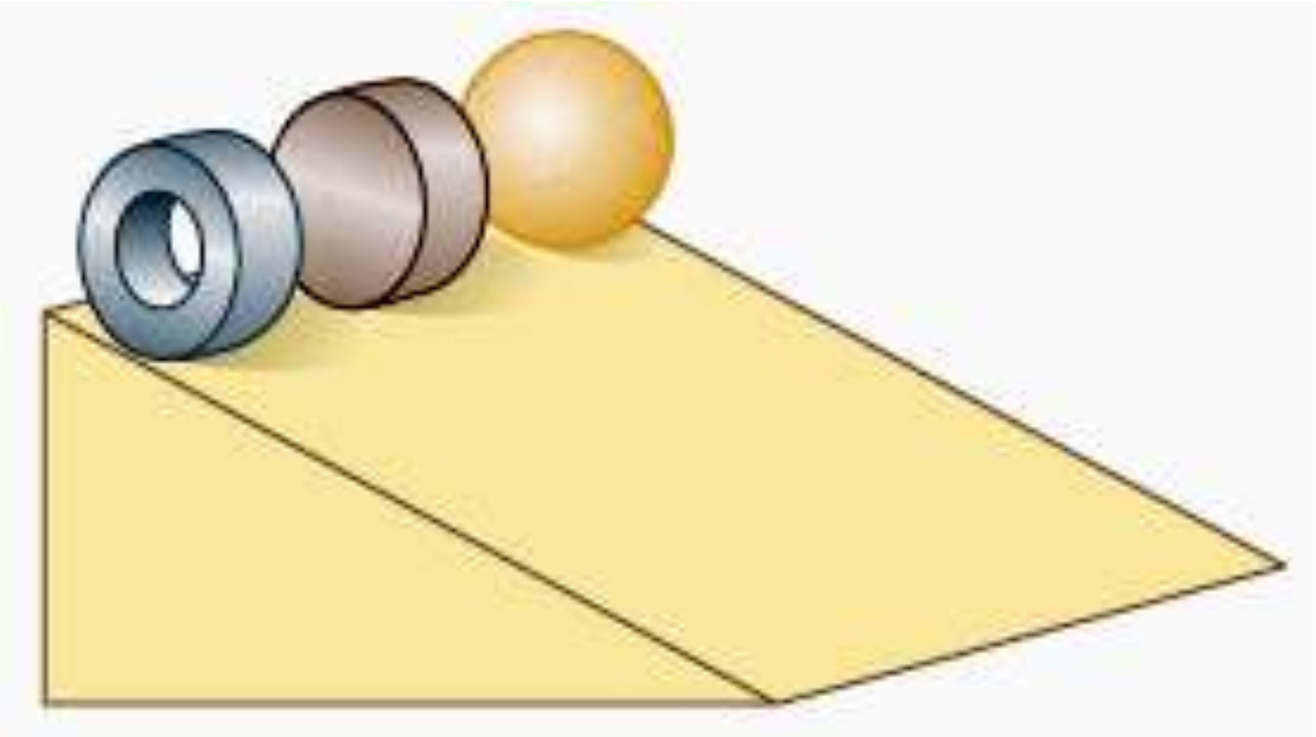
Problem

Four object are held in position at the corners of a rectangle by light rods as shown in figure. Find the moment of inertia of the system about (a) the *x*-axis, (b) the *y*-axis, and (c) an axis through O and perpendicular to the page.



Quiz (diskusi kelompok)

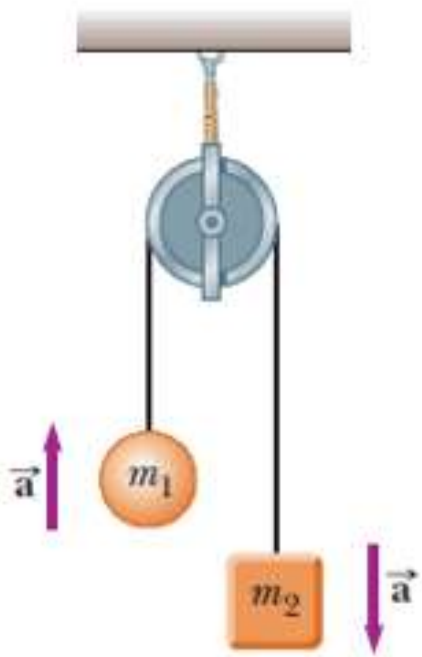
Three objects of uniform density—a solid sphere, a solid cylinder, and a hollow cylinder—are placed at the top of an incline. If they all are released from rest at the same elevation and roll without slipping, which object reaches the bottom first? Which reaches it last? You should try this at home and note that the result is independent of the masses and the radii of the objects.



Halliday, Resnick, Walker

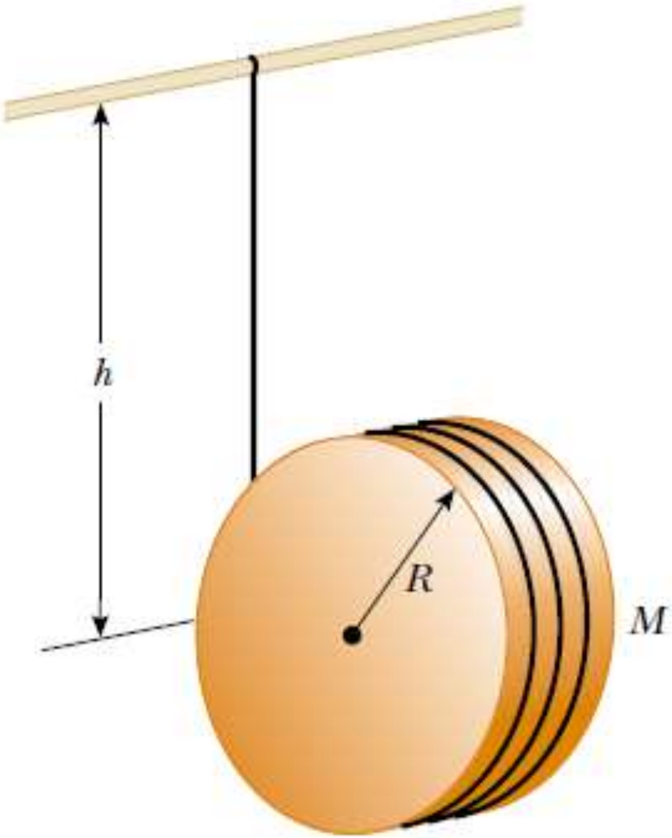
Problem

The pulley in Figure P8.59 has a moment of inertia of $5.0\ \text{kg}\cdot\text{m}^2$ and a radius of 0.50 m. The cord supporting the masses m_1 and m_2 does not slip, and the axle is frictionless. (a) Find the acceleration of each mass when $m_1 = 2.0$ kg and $m_2 = 5.0$ kg. (b) Find the tension in the cable supporting m_1 and the tension in the cable supporting m_2. [*Note:* The two tensions are different].



Soal

Seutas tali dililitkan pada sebuah cakram homogen berjari-jari R dan massa M. Cakram dilepaskan dari keadaan diam dengan tali vertikal dan ujung atasnya diikat pada sebuah palang tetap. Tentukan bahwa (a) tegangan talinya adalah sepertiga berat cakram, (b) besar percepatan pusat massanya adalah $2g/3$, dan (c) kelajuan pusat massanya adalah $(4gh/3)^{1/2}$ setalah cakram turun sejauh h. Buktikan jawaban (c) dengan menggunakan pendekatan energi.



$\sum F = T - Mg = -Ma$: $\sum \tau = TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$

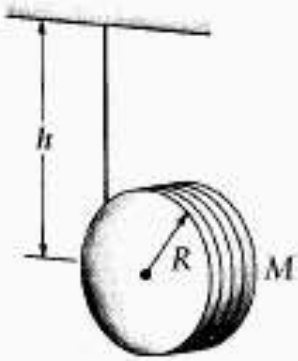


FIG. P10.77

(a) Combining the above two equations we find

$$T = M(g - a)$$

and

$$a = \frac{2T}{M}$$

thus $T = \boxed{\frac{Mg}{3}}$

(b) $a = \frac{2T}{M} = \frac{2}{M}\left(\frac{Mg}{3}\right) = \boxed{\frac{2}{3}g}$

(c) $v_f^2 = v_i^2 + 2a(x_f - x_i)$ $\quad v_f^2 = 0 + 2\left(\frac{2}{3}g\right)(h - 0)$

$$v_f = \boxed{\sqrt{\frac{4gh}{3}}}$$

For comparison, from conservation of energy for the system of the disk and the Earth we have

$U_{gi} + K_{\text{rot}\,i} + K_{\text{trans}\,i} = U_{gf} + K_{\text{rot}\,f} + K_{\text{trans}\,f}$: $\quad Mgh + 0 + 0 = 0 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f}{R}\right)^2 + \frac{1}{2}Mv_f^{\,2}$

$$v_f = \sqrt{\frac{4gh}{3}}$$

Tugas Merangkum:

1. Contoh 9.1 hal. 267
2. Contoh 9.2 hal. 268
3. Contoh 9.3 hal. 270
4. Contoh 9.7 hal. 274
5. Contoh 9.8 hal. 276
6. Contoh 9.13 hal 280
7. Contoh 10.4 hal. 297
8. Contoh 10.8 hal. 302

Pustaka: Young & Freedman, Fisika Universitas