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Analyzing Railguns with Excel: Simple Numerical Integration for the Classroom

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Any teacher of elementary physics can attest to the waning interest of students when all they are exposed to are boxes sliding down planes. In an effort to motivate interest, it is often useful to borrow examples from the real world. This way the students find it easy to identify the relevance of the subject matter. In addition, high-tech applications can be inspirational and serve to motivate the students to look deeper into the phenomena. One example of a technology that makes students sit up and pay attention are railguns. They are easily demonstrated using YouTube videos, especially now that the U.S. Navy has begun to release publicity videos of their latest designs[1] and associated fact sheet and papers.[2-4] This paper gives some background and simple models of the relevant physics of railguns.

A railgun uses the Lorentz magnetic force to accelerate projectiles,

$$\boldsymbol{F} = \boldsymbol{I}l \times \boldsymbol{B}, \qquad (1)$$

where $\boldsymbol{I}$ is the current, l is the length of the current element, and $\boldsymbol{B}$ is the magnetic field strength. It makes a natural subject of study for a secondary elementary physics class (electromagnetism). However, it also lends itself wonderfully to a primary elementary physics class (mechanics) because of its non-constant force. (Railguns are typically powered by banks of capacitors that result in a current and magnetic field which are not constant in time.) In this paper, we will first treat the railgun at the level of a mechanics course and then repeat with more electromagnetic theory included.

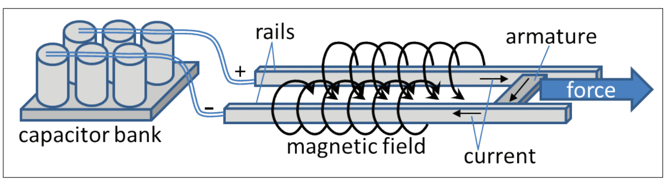

Fig. 1. Schematic of railgun.

Simple mechanics treatment

A schematic of a railgun is shown in Fig. 1. It consists of a bank of capacitors that supply current to two rails. The rails are bridged by an armature that carries the current between the rails and pushes the projectile. As the current flows along the rails, it creates a magnetic field. This magnetic field interacts with the current flowing along the armature to produce the Lorentz magnetic force. The armature is free to move and the magnetic force accelerates it along the rails. The quantity l in Eq. (1), therefore, is effectively the length of the armature (the distance between the rails).

In order to motivate the students, it is best to link calculations to a real-world railgun system. In a typical lesson, students were first shown the Navy publicity video in Ref. 1. From this video, several important pieces of information were estimated. First, it is a Mach 7 railgun; therefore, the final velocity of the projectile will be about seven times the speed of sound or about 2300 m/s. Second, the video shows that the rails are about 4 m long. Finally, the spokesperson holds up the projectile in one hand; therefore, the mass can be reasonably estimated as 5-10 kg.

In an entry-level mechanics course, problems are typically limited to those with constant acceleration. This restriction often causes the exclusion of interesting problems such as the railgun. However, this does not need to be the case. The simple equations that students memorize for constant acceleration problems can be used to create a numerical integration that allows the treatment of problems which do not have constant acceleration. This integration can be done in MATLAB for advanced students or even in Excel, as will be done here, for every type of student. For the current example, Newton's second law and the kinematic equations

$$a_{\text{old}} = F_{\text{old}}/m \qquad (2a)$$

$$v_{\text{new}} = v_{\text{old}} + a_{\text{old}}\,\Delta t \qquad (2b)$$

$$x_{\text{new}} = x_{\text{old}} + v_{\text{old}}\Delta t + \frac{a_{\text{old}}\Delta t^2}{2} \qquad (2c)$$

TITLE:	**Mach 7 Railgun**				
Mach	7				
Vgoal	2300	m/s	**F0**	1.00E+06	N
Lgoal	4	m	**mass**	10	kg
dt	0.002	s	**Tdecay**	0.5	s
Summary:	**Values at Vgoal of 2300 m/s:**				
24.0	95.3	2348	28.4		
(ms)	(km/s²)	(m/s)	(m)		
Calculated Iteration Values:					
	Eq. (2a)	Eq. (2b)	Eq. (2c)		
T	*A*	*V*	*X*		
(s)	(m/s²)	(m/s)	(m)		
0.0000	1.000E+05	0.0	0.000		
0.0020	9.960E+04	200.0	0.200		
0.0040	9.920E+04	399.2	0.799		
0.0060	9.881E+04	597.6	1.796		
0.0080	9.841E+04	795.2	3.189		
0.0100	9.802E+04	992.0	4.976		

Fig. 2. Portion of spreadsheet to calculate railgun performance.

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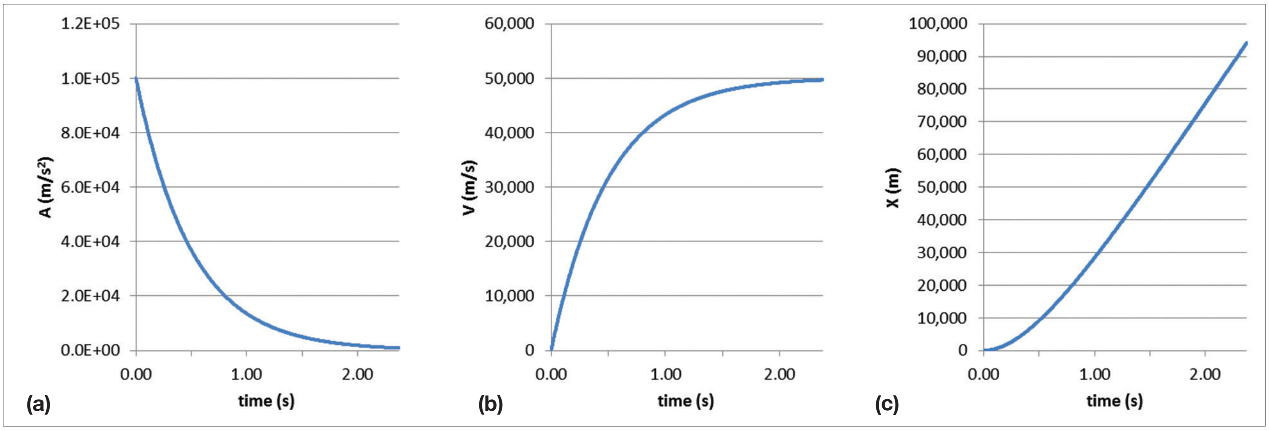

Fig. 3. Long-term behavior of the railgun with unrestricted length: (a) acceleration, (b) velocity, and (c) position.

will be used to update the position and velocity of the armature/projectile combination. In Eqs. (2), a is the acceleration, F is the magnetic force, m is the mass of the armature and projectile, v is the velocity, x is the position, and Δt is the time step. The subscripts "old" and "new" refer to the previous values and the updated values, respectively. In a formal numerical integration course, we call this Euler's method[5] and would analyze the error of these approximations, but, for the current purpose, we simply want to expose the student to the idea of a discretized integration. This is especially helpful to students who are learning the meaning of formal calculus integrals. These approximations assume that the acceleration is effectively constant over the small time steps.

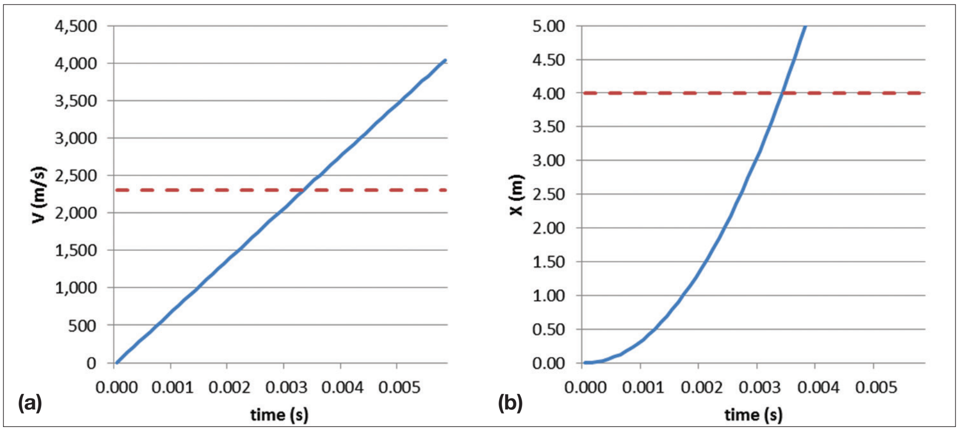

Fig. 4. Short-term behavior of a 4-m long railgun: (a) velocity and (b) position.

The initial force F_0 is not known, but it must decay because it is created by the current, which is decaying exponentially. For a simple mechanics treatment, the details of the force do not need to be specified and we asume it is given by

$$F = F_0 \exp(-t/t_0). \tag{3}$$

The time constant t_0 is typically set by the resistance and capacitance of the system. Since the objective is to limit the force drop as much as possible, engineers strive to make the time constant as long as possible. Thus, a design goal for this facet of the project is to select the time constant to be considerably longer than the flight time down the rails. This allows the gun to fire without having significant capacitor current drop as the projectile moves down the rails. Therefore, a reasonable estimate for the time constant is something on the order of 0.5 s. A deeper treatment of the force is given in the next section.

Given Eqs. (2)-(3), the students are asked to create a simulation of the naval railgun and discover what F_0 must be. The initial conditions are zero position and zero velocity with an unknown force. The final condition must be a velocity of 2300 m/s when the armature reaches the end of the 4-m gun. For the present example the mass m of the armature/projectile combination will be set to 10 kg. Figure 2 shows the first 10 milliseconds of the simulation. In the spreadsheet, each row uses the "old" values from the row above to calculate the "new" values using Eqs. (2) and (3). The initial force F_0 is not known and must be guessed. A nice feature of Excel is that once the first two rows are created with the proper referenced equations, they can be copied down to propagate the solution. In the present case the solution was propagated down 1000 cells (2 s) using a time step of 2 ms to explore the long-term behavior of the railgun.

The long-term behavior of the railgun, when the length of the rails is unrestricted and ignoring mechanical friction and aerodynamic drag, is shown in Fig. 3. As expected the acceleration decays to zero as the magnetic force goes to zero (because the current must go to zero as the capacitors drain). The velocity does not continue to increase, but instead asymptotes to about 50 km/s when the projectile has traveled more than 70 km. A real railgun is, of course, not this large and the computations were carried out this far simply to confirm the expected behavior. Simple numerical integration has allowed the students to be able to analyze a realistic non-constant acceleration example using their understanding of simple constant acceleration systems.

In order to "discover" the actual performance of our naval railgun, we will set the time step to 0.1 ms and increase the initial force F_0 until our performance goals are reached (2300 m/s at 4 m). A graph of this case is shown in Fig. 4. Increasing the initial force to 7×10^6 N causes the 10-kg projectile to experience nearly constant acceleration and to reach the end of the 4-m railgun in about 3.3 ms with a velocity near Mach 7 (2300 m/s). Note: it is not important that the goals are reached exactly since we are simply trying to estimate the performance of the railgun. Therefore, the initial force of the real-world naval railgun must be of the order of 7 million newtons or more than 1 million pounds! This is enough force to lift a fully loaded Boeing 747! The 10-kg projectile is subjected to more than 70,000 *g*'s! Students are frequently amazed by this "discovery" and the fact that they can learn something about a real-world system using nothing more than their constant acceleration tools combined with Excel.

More sophisticated electromagnetic treatment

The previous treatment was appropriate for an entry-level mechanics course. In this section, we will include more of the electromagnetic and circuits theory in order to explore the magnetic force. This section is appropriate for an entry-level class in electromagnetics.

With respect to electrical elements, a railgun has three elements: 1) resistive rails and wires, 2) capacitance, and 3) inductive rails. The inductance arises from the magnetic flux, which passes through the loop created by the rails and the armature. The electrical voltage drops in the railgun circuit can be written as

$$V_C - V_L - V_R = 0, \tag{4}$$

where V_C, V_L, and V_R are the voltage drops through the capacitor, inductor, and resistor, respectively. The voltage drop through the inductor needs special treatment because the inductance is changing as the armature moves down the rails and the magnetic flux increases. The energy in an inductor is given by

$$U_L = \frac{1}{2} L I^2, \tag{5}$$

where L is the inductance and I is the current. Differentiating with respect to time to get the power P in an inductor and using $P = IV_L$ allows the voltage drop in the inductor, V_L, to be written as

$$V_L = -\frac{I}{2}\frac{dL}{dt} - L\frac{dI}{dt}. \tag{6}$$

Using this result and the normal expressions for the voltage drop across a capacitor, $V_C = Q/C$, and resistor, $V_R = -IR$, allows Eq. (4) to be written as

$$\frac{Q}{C} + \frac{I}{2}\frac{dL}{dt} + L\frac{dI}{dt} + IR = 0, \tag{7}$$

where Q is the charge in the capacitor, C is the capacitance, and R is the resistance of the railgun. The current in the railgun can, therefore, be determined by using the following simple integration scheme:

$$\left(\frac{dI}{dt}\right)_{\text{old}} = \frac{-1}{L_{\text{old}}}\left(\frac{Q_{\text{old}}}{C} + \frac{I_{\text{old}}}{2}\frac{dL}{dt} + I_{\text{old}} R\right) \tag{8a}$$

$$I_{\text{new}} = I_{\text{old}} + \left(\frac{dI}{dt}\right)_{\text{old}} \Delta t \tag{8b}$$

$$Q_{\text{new}} = Q_{\text{old}} + I_{\text{old}} \Delta t \tag{8c}$$

$$\frac{dL}{dt} = \frac{(L_{\text{new}} - L_{\text{old}})}{\Delta t}, \tag{8d}$$

where the subscripts "new" and "old," again, refer to the current and previous time step, respectively. The inductance L is defined as

$$L = \frac{B_{\text{ave}} A}{I}, \tag{9}$$

where B_{ave} is the average magnetic field strength and A is the area subtended by the rails and the present armature location. The average magnetic field can be found formally by carrying out an integral of the Biot-Savart law or it can be estimated from Ampere's law. We will do the latter in this paper. Assuming that the rail separation is small compared to the length of the rails, the magnetic field at some distance R from the rail can be estimated from Ampere's law as

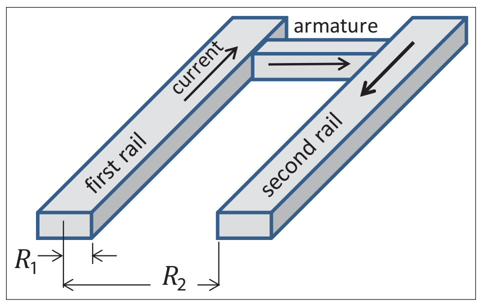

Fig. 5. Definition of limits for the calculation of average magnetic field.

$$B = \frac{\mu_0 I}{2\pi R}, \tag{10}$$

where μ_0 is the permeability of free space. Integrating from one rail to another and dividing by the rail separation yields the average magnetic field,

$$B_{\text{ave,2rails}} = \frac{\mu_0 I}{\pi(R_2 - R_1)} \ln(R_2 / R_1), \tag{11}$$

where R_1 and R_2 are defined as in Fig. 5. The result in Eq. (11) has been multiplied by two because the railgun utilizes a pair of rails. R_1 is the distance from the center of one rail to the rail surface and R_2 is the distance from the center of one rail to the start of the next rail. This assumes that all of the current is concentrated at the center of the rail. The real current is distributed throughout the entire rail area, but this would require a more involved integration. The average magnetic

TITLE: Mach 7 Rail Gun								
Capacitance (C)	1500	F	Voltage (t=0) (V_0)	875	V	mass	10	kg
Resistance (R)	2.00E-05	Ω	Charge (t=0) (Q_0)	-1.31E+06	Coul	Width	0.1	m
Inductance (t=0) (L_0)	2.00E-07	H	dL/dx	4.80E-07	H/m	μ_0	1.26E-06	N/A^2
dt	6.00E-07	s				R_1	0.01	m
						R_2	0.11	m

Summary: Values at Vgoal of 2300 m/s:									
3.8	-1.3E+06	3.1E+06	2.1E-06	-4.3E+08	4.7E+06	14.9	4.7E+05	2300.3	4.0
(ms)	(coul)	(Amp)	(H)	(A/s)	(N)	(T)	(m/s^2)	(m/s)	(m)

Calculated Iteration Values:									
T	Q	I	L	dI/dt	F	B_{avg}	a	v	x
(s)	(coul)	(Amp)	(H)	(A/s)	(N)	(T)	(m/s^2)	(m/s)	(m)
0.00E+00	-1.31E+06	0.00E+00	2.00E-07	4.38E+09	0.0	0.000	0.00	0.0	0.000
6.00E-07	-1.31E+06	2.63E+03	2.00E-07	4.38E+09	3.3	0.013	0.33	0.00E+00	0.00E+00
1.20E-06	-1.31E+06	5.25E+03	2.00E-07	4.37E+09	13.2	0.025	1.32	1.98E-07	5.95E-14
1.80E-06	-1.31E+06	7.87E+03	2.00E-07	4.37E+09	29.7	0.038	2.97	9.91E-07	4.16E-13
2.40E-06	-1.31E+06	1.05E+04	2.00E-07	4.37E+09	52.9	0.050	5.29	2.78E-06	1.55E-12
3.00E-06	-1.31E+06	1.31E+04	2.00E-07	4.37E+09	82.6	0.063	8.26	5.95E-06	4.16E-12

Fig. 6. Portion of spreadsheet to calculate railgun performance including electromagnetics.

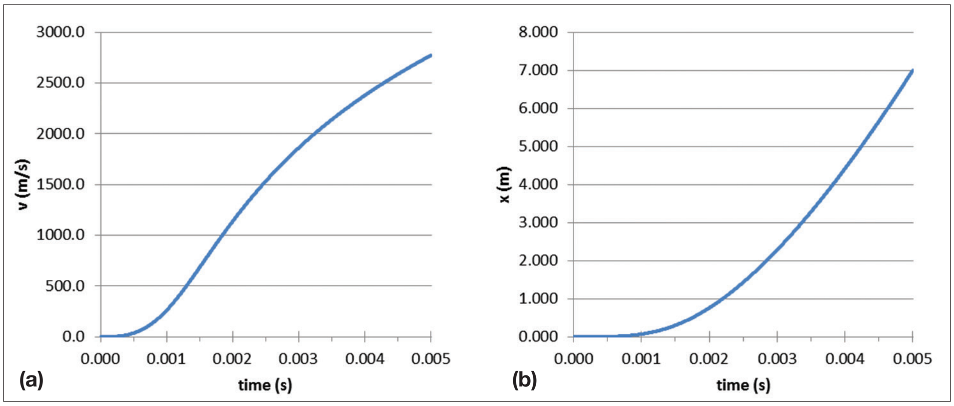

Fig. 7. eShort-term behavior of a 4-m long railgun including electromagnetics: (a) velocity and (b) position.

field is, therefore, sensitive to the value of R_1. A change in R_1 by a factor of 10, however, produces a change in B_{ave} of only a factor of two. After viewing the video of the naval railgun, the spacing between the rails, R_2, can be estimated as 0.1 m. R_1 is more difficult to estimate, but for the present example will be set to 0.01 m.

A full integration using the Biot-Savart law shows that the average magnetic field in Eq. (11) is too high by a factor of two (when the rails are longer than the separation between the rails). This is easy to explain from Fig. 5. Since we are interested in the average magnetic field at the location of the moving armature and the rails only have current *behind* the armature, the Ampere's law treatment should only approximate an infinite wire from *one* side. Therefore, the average magnetic field needs to be reduced by a factor of two. The average magnetic field at the location of the armature will be given by

$$B_{ave} = \frac{\mu_0 I}{2\pi(R_2 - R_1)} \ln(R_2 / R_1). \qquad (12)$$

Using this expression for the magnetic field, the inductance L in Eq. (9) can now be written as

$$L = \frac{\mu_0 \ln(R_2 / R_1)}{2\pi(R_2 - R_1)}(lx) + L_0, \qquad (13)$$

where l is the distance between the rails and x is the present location of the armature. The quantity L_0 will represent any inductance contained in the power system that is not a function of armature location. This may be due to the wires connecting the railgun and the capacitor banks and is discussed further below. With these last two expressions, all that is left to be determined is the force. Using Eqs. (1) and (12) gives the magnetic force as

$$F = \left[\frac{\mu_0 \ln(R_2 / R_1) l}{2\pi(R_2 - R_1)}\right] I^2. \qquad (14)$$

Note that the magnetic force scales with the square of the current. Therefore, the best way to generate large forces is to achieve high currents.

In reality we should include friction forces when simulating the railgun. Friction can easily be added to the simulation, but in the present example we will concentrate on the electromagnetic character of the railgun and ignore the friction between the armature and the rails.

The railgun can now be simulated using Eqs. (2), (8), (13), and (14). Creating an Excel spread sheet is straightforward. A section of it is shown in Fig. 6. In the present simulation, the resistance was estimated from aluminum rails 4 m long and 0.01 m square to be about 2×10^{-5} Ω. The quantity labeled "width" is the distance between the rails referred to as l above. The capacitance was increased until the results were no longer affected (about 1500 F). Also, an inductance L_0 was added because the actual system has some residual inductance in the power supply network. This was estimated to be about 10% of the total inductance of the 4-m rails, but this is a guess and must be measured on the real system. For consistent sign convention in Eqs. (7) and (8), the charge in the capacitor was initially set as negative. Setting the time step to a value of 6×10^{-7} s and copying down the rows several thousand cells allows the performance of the railgun to be estimated. In order to reach the performance goals (2300 m/s in 4 m), the voltage on the capacitors was increased using trial and error until the goal was met. The resulting velocity and position for the armature are shown in Fig. 7.

Comparing with Fig. 4 reveals that the velocity no longer rises quasi-linearly. Instead the acceleration (slope of the velocity vs. time) is initially small and then grows large before

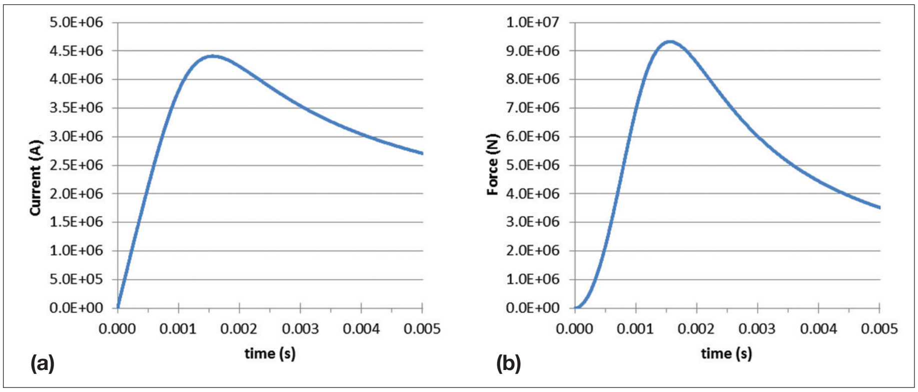

Fig. 8. Short-term behavior of a 4-m long railgun including electromagnetics: (a) current and (b) magnetic force.

diminishing again. An initial voltage on the capacitors of 875 V is required to achieve the desired performance. The reason for the behavior of the velocity/acceleration can be found in Fig. 8.

Figure 8 shows that inclusion of the draining capacitors, indeed, results in a current and, therefore, a force that decreases with time. The inclusion of the inductance, however, changes the behavior from a monotonically decreasing current and force to one that initially rises to a peak value and then begins the decay. Contrary to the simple treatment in the previous (mechanics only) section, the force starts at zero and then soars past even the previous estimate to over 9 million newtons or more than 2 million pounds! Our 10-kg projectile is, therefore, subjected to over 95,000 *g*'s! The current required to accomplish this is greater than 4 million amps.

Summary

We have analyzed an inherently nonlinear system, a railgun, using simple linear theory and Excel. The railgun has been treated at the level of a simple mechanics course and a more sophisticated electromagnetics course. Utilizing Excel for quasi-linear integration and a publicity YouTube video to estimate final performance, a real-world railgun was simulated. The mechanics treatment allowed the discovery of an estimate for the initial force, while the electromagnetic treatment displayed the full effects of the capacitors and inductive nature of this unique system. This activity has been found to be motivational and of great interest to entry-level physics students because it allows them to analyze a real-world system using the simple tools available to them. The activity has been found to be equally successful in advanced-level mechanics and numerical techniques courses.

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